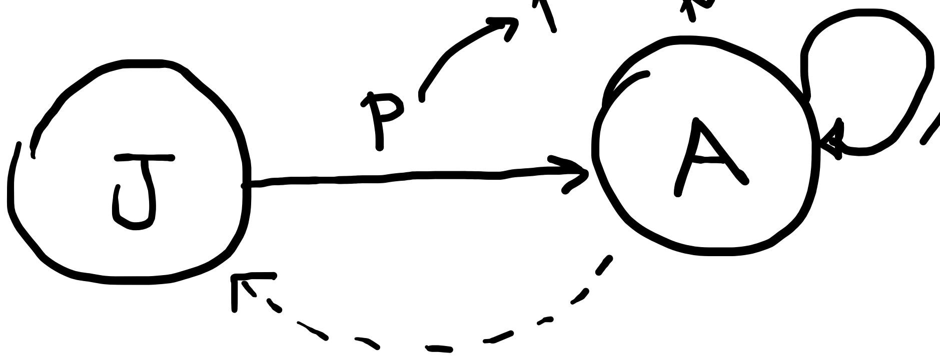
Populações estruturadas:

Juvenis e Adultos



fração de J que tornam adultos

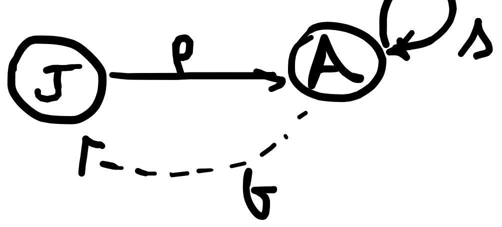
$s \to$ fração de adultos que permanecem adultos (sobrevivem)

$\beta \to$ nº de filhos por Adulto (fertilidade)

$$\begin{cases} J(n+1) = \beta A(n) \\ A(n+1) = pJ(n) + sA(n) \end{cases}$$

$$\begin{pmatrix} J \\ A \end{pmatrix}_{n+1} = \underbrace{\begin{pmatrix} 0 & \beta \\ p & s \end{pmatrix}}_{\mathbb{A}} \begin{pmatrix} J \\ A \end{pmatrix}_n$$

$b = 10.0$ $\quad$ $p = 0.1$ $\quad$ $s = 0.2$



n	$J(n)$	$A(n)$	Total(n)
0	0	1	1
1	10	0,2	10,2
2	2	1,04	3,04
⋮	⋮	⋮	⋮

⤳ Tabelas de vida

A → matriz de projeção, matriz de Leslie
matriz de transição, matriz de Lefkovitch

$$\begin{pmatrix} J \\ A \end{pmatrix}_{n+1} = \overbrace{\begin{pmatrix} 0 & b \\ p & s \end{pmatrix}}^{A} \begin{pmatrix} J \\ A \end{pmatrix}_{n}$$

$$\begin{pmatrix} J \\ A \end{pmatrix}_1 = \mathbb{A} \begin{pmatrix} J \\ A \end{pmatrix}_0$$

$$\begin{pmatrix} J \\ A \end{pmatrix}_2 = \mathbb{A} \begin{pmatrix} J \\ A \end{pmatrix}_1 = \mathbb{A} . \mathbb{A} \begin{pmatrix} J \\ A \end{pmatrix}_0 = \mathbb{A}^2 \begin{pmatrix} J \\ A \end{pmatrix}_0$$

$$\begin{pmatrix} J \\ A \end{pmatrix}_3 = \mathbb{A}^3 \begin{pmatrix} J \\ A \end{pmatrix}_0$$

$$p_{n+1} = R\, p_n$$

$$p_n = R^n p_0$$

autovalores e autovectores de $\mathbb{A}$ p/ encontrar a sol. geral para nossa prop de J e A.

$\det(A) \neq 0$ A possui autovetores e autovalores associados.

$$Av = \lambda v$$

$$\begin{pmatrix} 0 & b \\ p & s \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & b \\ p & s \end{pmatrix} \underbrace{\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}} - \lambda I \underbrace{\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}} = 0$$

$$\left[\begin{pmatrix} 0 & b \\ p & s \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

$$\underbrace{\begin{pmatrix} 0-\lambda & b \\ p & s-\lambda \end{pmatrix}}_{M} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = 0$$

p/ não ter somente a solução trivial, $\det M = 0$

$$(0-\lambda)(s-\lambda) - bp = 0$$

$$-\lambda s + \lambda^2 - bp = 0$$

$$\underbrace{\lambda^2 - \lambda s - bp} = 0$$

Pol caract.
Eq caract.

$\lambda^2 - \lambda s - bp = 0$ autovalores são os nossos $R_\pm$

$$\boxed{\lambda_\pm = \frac{s}{2} \pm \frac{1}{2}\sqrt{s^2 + 4bp}}$$

$$\lambda_\pm = \frac{0.2}{2} \pm \frac{1}{2}\sqrt{0{,}04 + 4 \cdot 10 \cdot 0{,}1} = 0.1 \pm \frac{1}{2}\sqrt{4.04}$$

$$\begin{cases} \lambda_+ = 1{,}05 \\ \lambda_- = -0{,}905 \end{cases}$$

Autovetores?

$$\begin{pmatrix} 0 & b \\ p & s \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$0.v_1 + bv_2 = \lambda v_1$$

$$bv_2 = \lambda v_1$$

$$v_2 = \frac{\lambda v_1}{b}$$

$$v_\pm = \begin{pmatrix} v_1 \\ \frac{\lambda v_1}{b} \end{pmatrix} = v_1 \begin{pmatrix} 1 \\ \frac{\lambda}{b} \end{pmatrix}$$

$$v_+ = v_1 \begin{pmatrix} 1 \\ \frac{1{,}05}{10} \end{pmatrix} \qquad v_- = v_1 \begin{pmatrix} 1 \\ \frac{-0.905}{10} \end{pmatrix}$$

$\lambda_+ = 1.05$ (autovalor dominante)

$$v_+ = \begin{pmatrix} 1 \\ 1{,}05/10 \end{pmatrix}$$

$$v_+ = \begin{pmatrix} 1 \\ \frac{1{,}05}{10} \end{pmatrix} = \frac{\begin{pmatrix} 1 \\ 0{,}105 \end{pmatrix}}{1{,}105} = \begin{pmatrix} 1/1{,}105 \\ \frac{0{,}105}{1{,}105} \end{pmatrix}$$

$\lambda_- = -0{,}905$

$$v_- = \begin{pmatrix} 1 \\ -0.905/10 \end{pmatrix}$$

$$P_n = \underbrace{C_+ \lambda_+^n}_{} \underbrace{\begin{pmatrix} 1 \\ 1{,}05/10 \end{pmatrix}}_{} + C_- \lambda_-^n \begin{pmatrix} 1 \\ \frac{-0.905}{10} \end{pmatrix}$$