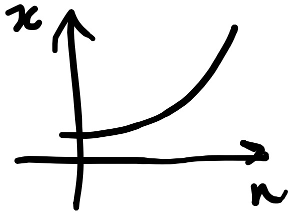
$$x_{n+1} = f(x_n, x_{n-1}, x_{n-2}, \ldots)$$

↳ f não é mais uma função linear

$$\boxed{x_{n+1} = \underbrace{R(x_n)}\, x_n}$$

$$x_{n+1} = \overset{=3}{R}\, x_n$$



1) Suponha que $R(x_n) = \dfrac{k}{b+x_n}$ k e b são ctes

$$x_{n+1} = \frac{k}{b+x_n}\, x_n = k x_n (b+x_n)^{-1}$$

2) Suponha que $R(x_n) = R_0\left(1 - \frac{x_n}{k}\right)$

$$x_{n+1} = R_0\left(1 - \frac{x_n}{k}\right)x_n = \left(R_0 - \frac{R_0 x_n}{k}\right)x_n = R_0 x_n - \frac{R_0}{k}x_n^2$$

3) Suponha que $R(x_n) = R_0 e^{-x_n/k}$

$$x_{n+1} = R_0 e^{-x_n/k} x_n$$

4) Suponha que $R(x_n) = \frac{R_0}{1+e^{a(x_n-k)}}$

$$x_{n+1} = \frac{R_0}{1+e^{a(x_n-k)}}x_n$$

1) $x_{n+1} = \dfrac{K}{b+x_n} x_n$ Modelos de saturação

situação que $x_n << b$

$$x_{n+1} = \overset{r}{\left(\frac{K}{b}\right)} x_n$$

pop que cresce exponencialmente

situação que $x_n >> b$

$$x_{n+1} = \frac{K}{\cancel{x_n}} \cancel{x_n}$$

$x_{n+1} \sim K$

Migração:

$$p_{n+1} = Rp_n + m \qquad p_{n+1} = p_n = \bar{p}$$

$$\bar{p} = R\bar{p} + m$$

$$\bar{p} - R\bar{p} = m \rightarrow \bar{p}(1-R) = m \rightarrow \boxed{\bar{p} = \frac{m}{1-R}}$$

1) $$x_{n+1} = \frac{k x_n}{b + x_n} \qquad x_{n+1} = x_n = \bar{x}$$

$$\bar{x} = \frac{k\bar{x}}{b+\bar{x}}$$

$$\bar{x}(b+\bar{x}) = k\bar{x}$$

$$\bar{x}(b+\bar{x}) - k\bar{x} = 0$$

$$\bar{x}(b+\bar{x}-k) = 0$$

$$\bar{x} = 0$$

$$b + \bar{x} - k = 0$$

$$\bar{x} = k - b$$

$$\bar{x} = 0$$

$$x_{n+1} = \frac{k x_n}{b + x_n} = \frac{k\,0}{b + 0} = 0$$

$$\bar{x} = k - b$$

$$x_{n+1} = \frac{k x_n}{b + x_n} = \frac{\cancel{k}(k - b)}{\cancel{b} + \cancel{k} - \cancel{b}} = k - b$$

2) $x_{n+1} = R_0\left(1 - \frac{x_n}{K}\right)x_n$

Mapa logístico

$x_{n+1} = x_n = \bar{x}$

$\bar{x} = R_0 \bar{x}\left(1 - \frac{\bar{x}}{K}\right)$

$\bar{x} = R_0\bar{x} - \frac{R_0 \bar{x}^2}{K}$

$\frac{R_0}{K}\bar{x}^2 - R_0\bar{x} + \bar{x} = 0$

$\underline{\underline{\bar{x}}}\left(\underline{\underline{\frac{R_0}{K}\bar{x} - R_0 + 1}}\right) = 0$

$\bar{x} = 0$ ✓

$\frac{R_0}{K}\bar{x} - R_0 + 1 = 0$

$\frac{R_0}{K}\bar{x} = R_0 - 1$

$\bar{x} = \frac{K}{R_0}(R_0 - 1)$

$x_{n+1} = R_0 x_n\left(1 - \frac{x_n}{K}\right)$

$\bar{x} = 0$

$x_{n+1} = R_0 . 0\left(1 - \frac{0}{K}\right) = 0$

$$x_{n+1} = R_0 x_n \left(1 - \frac{x_n}{K}\right)$$

$$\boxed{\bar{x} = \frac{K}{R_0}(R_0 - 1)}$$

$$x_{n+1} = \cancel{R_0} \frac{K}{\cancel{R_0}}(R_0 - 1)\left[1 - \frac{1}{\cancel{K}}\left(\frac{\cancel{K}}{R_0}(R_0 - 1)\right)\right]$$

$$= K(R_0 - 1)\left[1 - \frac{1}{R_0}(R_0 - 1)\right]$$

$$= K(R_0 - 1)\left[\frac{\cancel{R_0} - \cancel{R_0} + 1}{R_0}\right] = \underline{K(R_0 - 1)\frac{1}{R_0}}$$

3) $x_{n+1} = R_0 x_n e^{-x_n/k}$

$x_{n+1} = x_n = \bar{x}$

$\bar{x} = R_0 \bar{x} e^{-x/k}$

$\bar{x} - R_0 \bar{x} e^{-\bar{x}/k} = 0$

$\underbrace{\bar{x}}\,\underbrace{\left(1 - R_0 e^{-\bar{x}/k}\right)} = 0$

$\bar{x} = 0$

$1 - R_0 e^{-\bar{x}/k} = 0$

$1 = R_0 e^{-\bar{x}/k}$

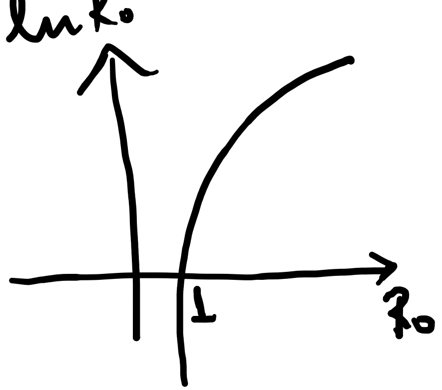
$\frac{1}{R_0} = e^{-\bar{x}/k}$

$\ln\left(\frac{1}{R_0}\right) = \ln e^{-\bar{x}/k}$

$\ln\left(\frac{1}{R_0}\right) = -\frac{\bar{x}}{k}$

$+\bar{x} = -k \underbrace{\ln\left(\frac{1}{R_0}\right)}_{\ln R_0^{-1}}$

$\bar{x} = k \ln R_0$



$$\bar{x} = k \ln R_0$$

$$x_{n+1} = R_0 x_n e^{-x_n/k}$$

$$= R_0 (k \ln R_0) e^{-\cancel{k} \ln R_0 / \cancel{k}}$$

$$= R_0 k \ln R_0 e^{-\ln R_0}$$

$$= \frac{R_0 k \ln R_0}{e^{\ln R_0}} = \frac{\cancel{R_0} k \ln R_0}{\cancel{R_0}} = k \ln R_0$$

4) $x_{n+1} = \dfrac{R_0}{1+e^{a(x_n-k)}} x_n$

$x_{n+1} = x_n = \bar{x}$

$\bar{x} = \dfrac{R_0 \bar{x}}{1+e^{a(\bar{x}-k)}}$

$\boxed{\bar{x} = 0}$

$\bar{x} - \dfrac{R_0 \bar{x}}{1+e^{a(\bar{x}-k)}} = 0$

$1 - \dfrac{R_0}{1+e^{a(\bar{x}-k)}} = 0$

$\bar{x}\underbrace{\left(1 - \dfrac{R_0}{1+e^{a(\bar{x}-k)}}\right)} = 0$

$\dfrac{R_0}{1+e^{a(\bar{x}-k)}} = 1$

$R_0 = 1 + e^{a(\bar{x}-k)}$

$\ln(R_0-1) = \ln e^{a(\bar{x}-k)}$

$\ln(R_0-1) = a(\bar{x}-k)$

$\ln(R_0-1) = a\bar{x} - ak$

$\ln(R_0-1) + ak = a\bar{x}$

$\underline{\bar{x} = \dfrac{1}{a}\ln(R_0-1) + k}$

Hassell (Varley e Grassel)

$$x_{n+1} = \underbrace{\lambda}_{\text{taxa de rep.}} x_n \underbrace{\left(\frac{1}{\alpha} x_n^{-b}\right)}_{\text{fração da prole que sobrevive até a fase adulta}}$$

$$x_{n+1} = x_n = \bar{x}$$

$$\bar{x} = \lambda \bar{x} \left(\frac{1}{\alpha} \bar{x}^{-b}\right)$$

$$\bar{x} - \lambda \bar{x} \left(\frac{1}{\alpha} \bar{x}^{-b}\right) = 0$$

$$\bar{x} \left(1 - \frac{\lambda}{\alpha} \bar{x}^{-b}\right) = 0$$

$$\boxed{\bar{x} = 0}$$

$$1 - \frac{\lambda}{\alpha} \bar{x}^{-b} = 0$$

$$\frac{\lambda}{\alpha} \bar{x}^{-b} = 1$$

$$\bar{x}^{-b} = \frac{\alpha}{\lambda}$$

$$\bar{x} = \left(\frac{\alpha}{\lambda}\right)^{-1/b}$$

$$\boxed{\bar{x} = \left(\frac{\lambda}{\alpha}\right)^{1/b}}$$

Modelo May

$$x_{n+1} = x_n e^{R(1 - x_n/K)}$$

Capacidade suporte K é um ponto fixo

$\bar{x} = x_{n+1} = x_n$

$$\bar{x} = \bar{x} e^{R(1-\bar{x}/K)}$$

$$\underline{\bar{x}} \left(\underline{1 - e^{R(1-\bar{x}/K)}}\right) = 0$$

$\underline{\bar{x} = 0}$ //

$$1 - e^{R(1-\bar{x}/K)} = 0$$

$$e^{R(1-\bar{x}/K)} = 1$$

$$\ln e^{R(1-\bar{x}/K)} = \ln 1$$

$$R\left(1 - \frac{\bar{x}}{K}\right) = 0$$

$$R - \frac{R\bar{x}}{K} = 0$$

$$\frac{R\bar{x}}{K} = R$$

$$\cancel{R}\bar{x} = K\cancel{R} \qquad \underline{\bar{x} = K} //$$

Desafio!

$$x_{n+1} = \lambda x_n \left(1 + \alpha x_n\right)^{-b}$$

Ache os pontos fixos deste modelo

generalização do modelo de saturação.

$$x_{n+1} = \frac{k x_n}{b + x_n}$$

$b = 1$

$\alpha = 1$