Aproximação WKB:

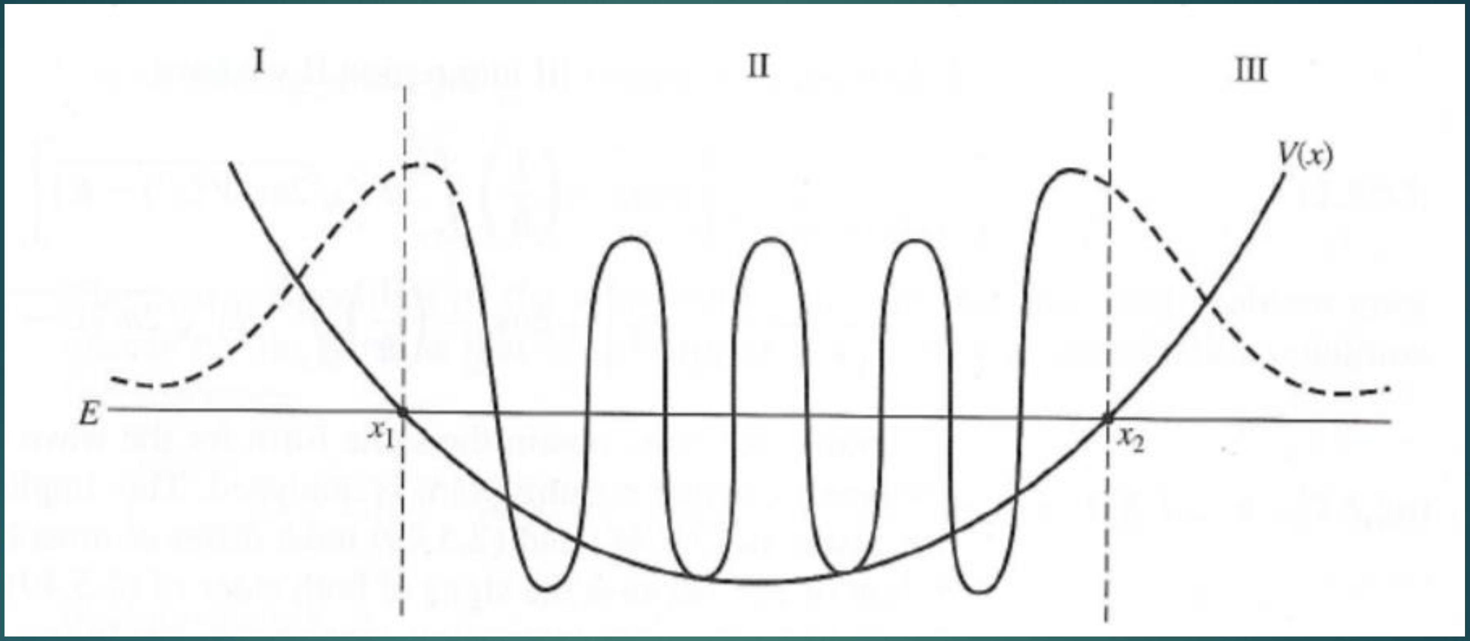
Oscilador Harmonico Simples

F 001 – MECÂNICA QUANTICA 1

PROF. MARCO AURÉLIO P. LIMA

ALUNO: ROBERTO DE OLIVEIRA ZURITA

Quantização dos níveis de energia

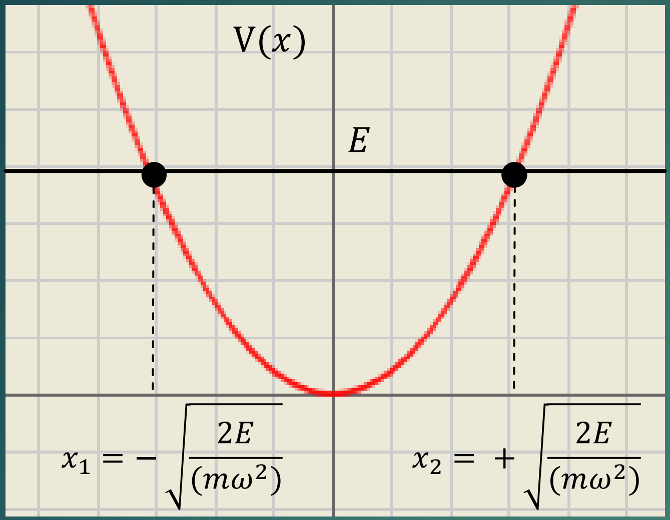


Região II

$$\begin{cases} \Psi_{II}^{1} = \dfrac{2}{(E - V(x))^{1/4}} \cos\left[\dfrac{1}{\hbar}\displaystyle\int_{x_1}^{x} \sqrt{2m(E - V(x'))}dx' - \dfrac{\pi}{4}\right] \\ \Psi_{II}^{2} = \dfrac{2}{(E - V(x))^{1/4}} \cos\left[-\dfrac{1}{\hbar}\displaystyle\int_{x}^{x_2} \sqrt{2m(E - V(x'))}dx' + \dfrac{\pi}{4}\right] \end{cases}$$

$$A \cos(a) = B \cos(b) \;\Rightarrow\; a - b = n\pi \quad \Rightarrow \quad \int_{x_1}^{x_2} \sqrt{2m(E - V(x'))}dx' = \left(n + \frac{1}{2}\right)\pi\hbar$$

Potencial harmônico



$$V(x) = \frac{1}{2}\, m\omega^2 x^2$$

$$\int_{x_1}^{x_2} \sqrt{2m\left(E - \frac{1}{2}\, m\omega^2 x^2\right)}\, dx' = 2m\omega \int_{0}^{x_2} \sqrt{\frac{2E}{m\omega^2} - \, x^2 dx'} =$$

Fazendo $x_2 = \sqrt{\frac{2E}{(m\omega^2)}}$ *e realizando a mudança de variável* $x = x_2 \sin(\theta)$ *com* $\theta \in [0, \pi/2]$

$$= 2\, m\, \omega\, x_2^2 \int_0^{\pi/2} \cos^2(\theta) d\theta = m\, \omega\, x_2^2 \int_0^{\pi/2} (1 + \cos(2\theta)) d\theta = m\, \omega\, x_2^2 \left(\frac{\pi}{2}\right) =$$

$$= \frac{\pi E}{\omega} = \left(n + \frac{1}{2}\right)\pi\hbar \Rightarrow \boxed{E = \left(n + \frac{1}{2}\right)\hbar\omega}$$