# Aula 17

Exercício 21.3 $\quad [N_i, N_j] = 0$ (mostre)

$$N_i N_j \, |n_1 n_2 \dots n_i \dots n_j\rangle = n_i n_j \, | \quad \rangle$$
$$N_j N_i \, | \quad \rangle = n_j n_i \, | \quad \rangle \qquad \text{cqd}$$

Exercício 21.4 $\quad$ ($n_i = 0$ ou $1$ p/ fermions e $n_i \geq 0$ e inteiro para bósons)

$$a_i a_i^+ - a_i^+ a_i = 1$$

$$\langle n_1 \dots n_i \dots | a_i a_i^+ - a_i^+ a_i | n_1 \dots n_i \dots \rangle = 1$$

$$a_i^+ |\underline{0}\rangle = \psi_i^{(1)} = |0 \dots n_i = 1 \dots\rangle$$

$$a_i^{+2} |\underline{0}\rangle = 0 \qquad \text{(caso fermions)}$$

$$a_i^{+2} |\underline{0}\rangle = |0 \dots n_i = 2 \dots\rangle$$

$$N_i a_i^+ = a_i^+ (N_i + 1)$$

$$N_i a_i^+ |\dots n_i \dots\rangle = (n_i + 1) a_i^+ |\dots n_i \dots\rangle$$

$\therefore a_i^+ |\dots n_i \dots\rangle$ é autoket de $N_i$ com autovalor $(n_i + 1)$

$$N_i a_i^{+2} = a_i^+ (N_i + 1) a_i^+ = a_i^{+2} (N_i + 2)$$

$$N \underbrace{a_i^{+2} |\dots n_i \dots\rangle}_{\text{autoket}} = a_i^{+2} (n_i + 2) |\dots n_i \dots\rangle = (n_i + 2) a_i^{+2} |\dots n_i \dots\rangle$$

autoket c/ autovalor $(n_i + 2)$

Que é inteiro → suponha que não é e aplique $a_i$ até passar o zero ($n_i$ fica negativo)

e não pode pois $a_i |n_1 \dots n_i \dots\rangle$ tem norma positiva

$$\langle n_1 \dots n_i \dots | a_i^+ a_i | n_1 \dots n_i \dots\rangle = n_i > 0$$

$N_i = a_i^\dagger a_i$

$N_i |n_1 n_2 \dots n_i \dots\rangle = n_i |n_1 n_2 \dots n_i \dots\rangle$
c/ $n_i \geq 0$

$a_i |n_1 n_2 \dots n_i \dots\rangle = c |n_1 n_2 \dots n_i - 1 \dots\rangle$

$\downarrow$

norma

$\langle \dots n_i \dots | a_i^\dagger a_i | \dots n_i \dots \rangle = n_i = |c|^2$

$\therefore \quad c = e^{i\alpha} \sqrt{n}$

p/ Bose-Einstein $\quad \alpha = 0$

p/ Fermi-Dirac $\begin{cases} e^{i\alpha} = +1 & \text{se \# de estados ocupados anteriores for par} \\ e^{i\alpha} = -1 & \text{se for ímpar} \end{cases}$

$a_1 |111\dots\rangle = |0111\rangle$

$a_2 |011\dots\rangle = -|0011\rangle$

$\therefore \quad a_2 a_1 |111\dots\rangle = |0011\rangle$

$a_2 |111\dots\rangle = -|101\rangle$

$a_1 |101\rangle = |0011\rangle$

$a_1 a_2 |111\dots\rangle = -|0011\rangle$

$(a_1 a_2 + a_2 a_1) |111\rangle = 0$

Resumo

$a^\dagger a |n\rangle = n |n\rangle$

BE (~ oscilador)

$a|n\rangle = \sqrt{n} |n-1\rangle$

$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$

FD

$a|0\rangle = 0; \ a|1\rangle = e^{i\alpha} |0\rangle$

$a^\dagger |0\rangle = e^{-i\alpha} |1\rangle; \ a^\dagger |1\rangle = 0$

# Variáveis Dinâmicas

$\mathbb{K}$ operador que mede valor total de uma quantidade aditiva K de uma partícula

$$\mathbb{K} = \sum_i K_i N_i = \sum_i K_i a_i^+ a_i$$

lembrando $\quad a_i^+ = \sum_\ell b_\ell^+ \langle L_\ell | K_i \rangle$

$$a_i = \sum_k b_k \langle K_i | L_k \rangle$$

$$\mathbb{K} = \sum_{i\ell k} b_\ell^+ b_k \langle L_\ell | K_i \rangle K_i \langle K_i | L_k \rangle$$

$$= \sum_{\ell k} b_\ell^+ b_k \sum_i \langle L_\ell | K | K_i \rangle \langle K_i | L_k \rangle$$

$$\therefore \quad \mathbb{K} = \sum_{\ell k} b_\ell^+ b_k \langle L_\ell | K | L_k \rangle$$

operador de duas partículas (ainda aditivo)

$$\mathbb{V} = \underbrace{\frac{1}{2} \sum_{i \neq j} N_i N_j V_{ij}}_{\substack{\text{interação} \\ \text{entre blocos} \\ \text{distintos}}} + \underbrace{\frac{1}{2} \sum_i N_i (N_i - 1) V_{ii}}_{\substack{\text{interação} \\ \text{dentro de} \\ \text{cada bloco}}}$$

$C_{N,2} = \frac{N!}{(N-2)!\,2!}$

$$V = \frac{1}{2}\sum_{ij} \underbrace{(N_i N_j - N_i \delta_{ij})}_{P_{ij}} V_{ij}$$

Operador de distribuição de pares

$$P_{ij} = N_i N_j - N_i \delta_{ij} = a_i^+ a_i a_j^+ a_j - a_i^+ a_i \delta_{ij}$$

$$a_i a_j^+ \mp a_j^+ a_i = \delta_{ij}$$

$$= a_i^+ (\delta_{ij} \pm a_j^+ a_i) a_j - a_i^+ a_i \delta_{ij}$$

$$= \pm a_i^+ a_j^+ a_i a_j = (\pm)(\pm)\, a_i^+ a_j^+ a_j a_i = a_i^+ a_j^+ a_j a_i$$

$$a_i a_j \mp a_j a_i = 0$$

$$\therefore\ V = \frac{1}{2}\sum_{ij} a_i^+ a_j^+ a_j a_i V_{ij}$$

transformação $\quad a_i^+ = \sum_q b_q^+ \langle L_q | K_i \rangle$

$$a_j = \sum_s b_s \langle K_j | L_s \rangle$$

$$V = \frac{1}{2}\sum_{\substack{qrst\\ ij}} b_q^+ b_r^+ b_s b_t \langle L_q|K_i\rangle \langle L_r|K_j\rangle \langle K_j|L_s\rangle \langle K_i|L_t\rangle \cdot V_{ij}$$

$$\equiv \frac{1}{2}\sum_{qrst} b_q^+ b_r^+ b_s b_t \langle qr|V|ts\rangle$$

onde $\langle qr|V|ts\rangle = \sum_{ij} \underbrace{\langle L_q|K_i\rangle \langle K_i|L_t\rangle}_{\text{1 partícula}} \underbrace{\langle L_r|K_j\rangle \langle K_j|L_s\rangle}_{\text{outra partícula}} V_{ij}$

note que

$$\sum_{ij} \langle L_q | K_i \rangle \langle L_r | K_j \rangle \langle K_j | L_s \rangle \langle K_i | L_t \rangle V_{ij}$$

$$= \sum_{ij} \langle L_q L_r | K_i K_j \rangle V_{ij} \langle K_i K_j | L_t L_s \rangle$$

$$= \sum_{ij} \langle L_q L_r | V | K_i K_j \rangle \langle K_i K_j | L_t L_s \rangle$$

$$= \langle L_q L_r | V \left( \sum_{ij} | K_i K_j \rangle \langle K_i K_j | \right) | L_t L_s \rangle$$

$$= \langle L_q L_r | V | L_t L_s \rangle = \langle q r | V | t s \rangle$$

Exercícios 21.6, 21.7 e 21.8